Common Logarithms and Their Properties

Earthquakes and Richter Scale

2.4

In 1935, Charles F. Richter developed the Richter magnitude test scale to compare the size of earthquakes. The Richter scale is based on the amplitude of the seismic waves recorded on seismographs at various locations after being adjusted for distance from the epicenter of the earthquake.

Richter assigned a magnitude of 0 to a standard earthquake, whose amplitude on a seismograph is 1 micron, or $10^{-4}$ cm. According to the Richter scale, a magnitude-1.0 earthquake causes 10 times the ground motion of a standard earthquake. A magnitude-2.0 earthquake causes 10 times the ground motion of a magnitude-1.0 earthquake. This pattern continues as the magnitude of the earthquake increases.

1. How does the ground motion caused by earthquakes of these magnitudes compare?

a. magnitude-5.0 earthquake compared to magnitude 4.0

10 times as much

b. magnitude-4.0 earthquake compared to magnitude 1.0

1000 times as powerful

c. magnitude-4.0 earthquake compared to a standard earthquake whose magnitude is 0

10,000 times

2. The table below describes the effects of different magnitude earthquakes. Complete the table to show how many times as great the ground motion is when caused by each earthquake as compared to the standard earthquake.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Ground Motion Compared to Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2.0</td>
<td>100</td>
</tr>
<tr>
<td>3.0</td>
<td>1,000</td>
</tr>
<tr>
<td>4.0</td>
<td>10,000</td>
</tr>
<tr>
<td>5.0</td>
<td>100,000</td>
</tr>
</tbody>
</table>

Typical Effects of Earthquakes of Various Magnitudes

1.0 Very weak, no visible damage
2.0 Not felt by humans
3.0 Often felt, usually no damage
4.0 Windows rattle, indoor items shake
5.0 Damage to poorly constructed structures, slight damage to well-designed buildings
6.0 Destructive in populated areas
7.0 Serious damage over large geographic areas
8.0 Serious damage across areas of hundreds of miles
9.0 Serious damage across areas of hundreds of miles
10.0 Extremely rare, never recorded
3. In Parts a–c, you will graph the data from Item 2. Let the horizontal axis represent the magnitude of the earthquake and the vertical axis represent the amount of ground motion caused by the earthquake as compared to a standard earthquake. Use the grid in the My Notes space or a separate sheet of grid paper.

\[ \text{a. Plot the data, using a grid that displays } -10 \leq x \leq 10 \text{ and } -10 \leq y \leq 10. \text{ Explain why this grid is or is not a good choice.} \]

\[ \text{b. Plot the data, using a grid that displays } -10 \leq x \leq 100 \text{ and } -10 \leq y \leq 100. \text{ Explain why this grid is or is not a good choice.} \]

\[ \text{c. A scale may be easier to choose if only a subset of the data is graphed. Determine an appropriate subset of the data and a scale for the graph. Draw the graph and label and scale the axes.} \]

\[ \text{d. Draw a function that fits the data plotted on the graph in Part c. Write a function } G \text{ for the ground motion caused compared to a standard earthquake by a magnitude-}x \text{ earthquake.} \]
Charles Richter needed a way to convert these values into more accessible numbers. He also realized that the magnitude of an earthquake is a function of ground motion caused by the earthquake, because only after the ground motion is measured by a seismograph can a magnitude be assigned to the earthquake.

4. In Item 3, the data was plotted so that the ground motion caused by the earthquake was a function of the magnitude of the earthquake.

   a. Is the ground motion a result of the magnitude of an earthquake or is the magnitude of an earthquake the result of ground motion?

   b. Based your answer to Part a, would you choose ground motion or magnitude as the independent variable of a function relating the two quantities? What would you choose as the dependent variable?

   c. Make a new graph of the data plotted Item 3c so that the magnitude of the earthquake is a function of the ground motion caused by the earthquake. Scale the axes and draw a function that fits the plotted data. Use the grid in the My Notes space or use a separate sheet of grid paper.

5. Let the function you graphed in Item 4c be \( y = M(x) \), where \( M \) is the magnitude of an earthquake that causes \( x \) times as much ground motion as a standard earthquake.

   a. State the domain and the range of the function \( y = G(x) \) from Item 3d and the function \( y = M(x) \).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = G(x) )</td>
<td>( y = M(x) )</td>
</tr>
<tr>
<td>Domain</td>
<td>Range</td>
</tr>
</tbody>
</table>
5. (continued)

b. In terms of the problem situation, describe the meaning of an ordered pair on the graphs of \( y = G(x) \) and \( y = M(x) \).

\[
\begin{align*}
y &= G(x) & (\quad , \quad ) \\
y &= M(x) & (\quad , \quad )
\end{align*}
\]

c. A portion of the graphs of \( y = G(x) \) and \( y = M(x) \) is shown on the same set of axes. Describe any patterns you observe.

![Graph of G(x) and M(x)]


d. What is the relationship between the functions \( G \) and \( M \)?
The Richter scale uses a base 10 logarithmic scale. A base 10 logarithmic scale means that when the ground motion is expressed as a power of 10, the magnitude of the earthquake is the exponent. You have seen this function $G(x) = 10^x$, where $x$ is the magnitude, in Item 3d.

The function $M$ is the inverse of an exponential function $G$ whose base is 10. The algebraic rule for $M$ is a common logarithmic function. Write this function as $M(x) = \log x$, where $x$ is the ground motion compared to a standard earthquake.

6. Graph $M(x) = \log x$ on a graphing calculator.

   a. Sketch the calculator graph on the grid in the My Notes space. Be certain to label and scale each axis.

   b. Use $M$ to estimate the magnitude of an earthquake that causes 120,000 times the ground motion of a standard earthquake. Describe what would happen if this earthquake was centered underneath a large metropolitan city.

   c. Use $M$ to determine the amount of ground motion caused by the 2002 magnitude-7.9 Denali earthquake.
7. Complete the tables below to show the relationship between the exponential function base 10 and its inverse, the common logarithmic function.

<table>
<thead>
<tr>
<th>x</th>
<th>y = 10^x</th>
<th>x</th>
<th>y = log x</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10^0 = 1</td>
<td>1</td>
<td>log 1 = 0</td>
</tr>
<tr>
<td>1</td>
<td>10^1</td>
<td>10</td>
<td>log 10^1</td>
</tr>
<tr>
<td>2</td>
<td>10^2</td>
<td>100</td>
<td>log 10^2</td>
</tr>
<tr>
<td>3</td>
<td>10^3</td>
<td>1000</td>
<td>log 10^3</td>
</tr>
<tr>
<td>log x</td>
<td></td>
<td>10^x</td>
<td></td>
</tr>
</tbody>
</table>

8. Use the information in Item 7 to write a logarithmic statement for each exponential statement.
   a. 10^4 = 10,000
   b. 10^{-1} = \frac{1}{10}

9. Use the information in Item 7 to write each logarithmic statement as an exponential statement.
   a. log 100,000 = 5
   b. log \left(\frac{1}{100}\right) = -2

10. Evaluate each logarithmic expression without using a calculator.
   a. log 1000
   b. log \left(\frac{1}{10,000}\right)
Common Logarithms and Their Properties
Earthquakes and Richter Scale

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Create Representations

You have already learned the properties of exponents. Logarithms also have properties.

11. Complete these three properties of exponents.

\[ a^m \cdot a^n = \] 

\[ \frac{a^m}{a^n} = \] 

\[ (a^m)^n = \] 

12. Use a calculator to complete the tables below. Round each answer to the nearest thousandth.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \log x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \log x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

13. Add the logarithms from the tables in Item 12 to see if you can develop a property. Find each sum and round each answer to the nearest hundredth.

\[ \log 2 + \log 3 = \] 

\[ \log 2 + \log 4 = \] 

\[ \log 2 + \log 5 = \] 

\[ \log 3 + \log 3 = \]
14. Compare the answers in Item 13 to the tables of data in Item 12.

a. Is there a pattern or property when these logarithms are added? If yes, explain the pattern that you have found.

b. State the property of logarithms that you found by completing the following statement.

\[ \log m + \log n = \] ________________

15. Explain the connection between the property of logarithms stated in Item 14 and the corresponding property of exponents in Item 11.

16. Graph \( y_1 = \log 2 + \log x \) and \( y_2 = \log 2x \) on a graphing calculator. What do you observe? Explain.

17. Make a conjecture about the property of logarithms that relates to the property of exponential equations that states the following:

\[ \frac{a^m}{a^n} = a^{m-n}. \]
18. Use the information from the tables in Item 12 to provide examples that support your conjecture in Item 17.

19. Graph \( y_1 = \log x - \log 2 \) and \( y_2 = \log \left( \frac{x}{2} \right) \) on a graphing calculator. What do you observe?

20. Make a conjecture about the property of logarithms that relates to the property of exponents that states the following: \((a^m)^n = a^{mn}\).

21. Use the information from the tables in Item 12 and the properties developed in Items 14 and 17 to support your conjecture in Item 20.

22. Graph \( y_1 = 2 \log x \) and \( y_2 = \log x^2 \) on a graphing calculator. What do you observe?

23. The logarithmic properties that you conjectured and then verified in Items 14, 17, and 20 are listed below. State each property.

   **Product Property:** __________________________

   **Quotient Property:** __________________________

   **Power Property:** __________________________
My Notes

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SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Create Representations

24. Use the properties from Item 23 to condense each expression to a single logarithm. Assume all variables are positive.
   a. \( \log x - \log 7 \)
   b. \( 2 \log x + \log y \)

25. Use the properties from Item 23 to expand each expression. Assume all variables are positive.
   a. \( \log 5xy^4 \)
   b. \( \log \frac{x}{y^3} \)

26. Condense each expression to a single logarithm. Then evaluate.
   a. \( \log 2 + \log 5 \)
   b. \( \log 5000 - \log 5 \)
   c. \( 2 \log 5 + \log 4 \)

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Let \( f(x) = 10^x \) and let \( g(x) = f^{-1}(x) \). What is the algebraic rule for \( g(x) \)?
2. Evaluate without using a calculator.
   a. \( \log 10^6 \)
   b. \( 10^{0.4} \)
   c. \( \log 1,000,000 \)
   d. \( \log \frac{1}{100} \)
3. Write an exponential statement for each.
   a. \( \log 10 = 1 \)
   b. \( \log \frac{1}{1,000,000} = -6 \)
   c. \( \log a = b \)
4. Write a logarithmic statement for each.
   a. \( 10^7 = 10,000,000 \)
   b. \( 10^0 = 1 \)
   c. \( 10^m = n \)
5. Condense each expression to a single logarithm. Then evaluate the expression without using a calculator.
   a. \( \log 5 + \log 20 \)
   b. \( \log 3 - \log 30 \)
   c. \( 2 \log 400 - \log 16 \)
   d. \( \log \left( \frac{1}{400} \right) + 2 \log 2 \)
6. MATHEMATICAL REFLECTION Explain why \( \log (-100) \) is not defined.